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The little Higgs from a simple group

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Abstract: We present a model of electroweak symmetry breaking in which the Higgs boson is a pseudo-Nambu-Goldstone boson. By embedding the standard models SU(2) $\times$ U(1) into an SU(4) $\times$ U(1) gauge group, one-loop quadratic divergences to the Higgs mass from gauge and top loops are canceled automatically with the minimal particle content. The potential contains a Higgs quartic coupling which does not introduce one-loop quadratic divergences. Our theory is weakly coupled at the electroweak scale, it has new weakly coupled particles at the TeV scale and a cutoff above 10 TeV, all without fine tuning. We discuss the spectrum of the model and estimate the constraints from electroweak precision measurements.

Keywords: Beyond Standard Model, Higgs Physics.

"He who hath clean hands and a good heart is okay in my book,
but he who fools around with barnyard animals has got to be watched"
W. Allen
"The littler the Higgs the bigger the group"
S. Glashow

Contents

1. Introduction

The Standard Model (SM) is well supported by all high energy data [1]. Precision tests match predictions including one-loop quantum corrections. This suggest the SM is a valid description of Nature up to energies in the multi-TeV range with a Higgs mass which is less than about 200 GeV [2].

This picture is not satisfying because the Higgs gets one-loop quadratically divergent corrections to its squared mass. The most significant corrections come from loops of top quarks, W-bosons and the Higgs (figure 1). The top loop is the most severe - demanding a contribution to the Higgs mass of 200 GeV or less requires a momentum cutoff $\Lambda_{\rm top} \lesssim$ 700 GeV. If one allows fine tuning of order 10% between this correction and a counter term, one still needs $\Lambda_{\rm top} \lesssim 2$ TeV.

The need to cancel quadratic divergences along with the consistency of electroweak precision measurements with the SM suggests new *weakly coupled* physics at ~ 1 TeV.

Supersymmetry softly broken around 1 TeV is an example of new physics that meets these criteria. Loops of superpartners cancel all quadratic divergences in the SM. The minimal supersymmetric standard model's most compelling feature is its suggestive unification of couplings. Its least compelling feature is the fact that it has over 100 new parameters and yet the current bound on the Higgs mass requires fine-tuning of parameters for successful electroweak symmetry breaking. Models that improve this situation exist, and thus weakly coupled superpartners at the weak scale remains an interesting option. However, as we close in on the parameter space of the minimal supersymmetric standard model, it is of great interest to find alternative weakly coupled theories of electroweak symmetry breaking.

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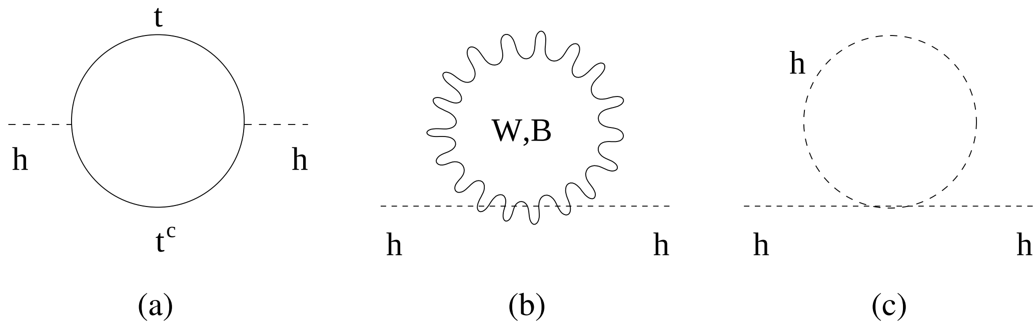


Figure 1: Quadratically divergent contributions to the Higgs mass in the Standard model: top loop, $\mathrm{SU}(2) \times \mathrm{U}(1)$ gauge boson loops and Higgs loop.

A new class of models, called "Little Higgs" theories [3]–[8], produce a light Higgs boson with weakly coupled physics up to 10s of TeV. As in composite Higgs models [9], the Higgs is a pseudo-Nambu-Goldstone boson (PNGB) and is massless at tree-level. Its mass is protected by a global symmetry which is spontaneously broken. The symmetry is explicitly broken by weakly coupled operators in the theory which become the normal SM couplings below 1 TeV. The Little Higgs trick is that no one operator alone explicitly breaks the global symmetry protecting the Higgs mass, and therefore no quadratically divergent contribution exists at one loop. The purpose of this article is to present a new model of this type with the simplest gauge group to date.

In the next section, we show that simply extending the electroweak gauge group to $\mathrm{SU}(3) \times \mathrm{U}(1)$ *automatically* removes one-loop quadratic divergences from the gauge and top loops. We'll show that if two scalar triplets have vacuum expectation values (VEVs) in the same direction, breaking the gauge group to $\mathrm{SU}(2) \times \mathrm{U}(1)$, there exists an SU(2) doublet whose mass is protected from quadratic divergences at one loop. Adding the minimum (weakly coupled) top sector to generate a Yukawa coupling automatically protects the Higgs mass at one loop. The key is that by having both triplets get VEVs, both produce a set of NGBs (including doublets), either of which could have been eaten by the massive gauge fields. It is the fact that no one operator in the tree-level lagrangian contains both triplets which protects the doublet mass. The symmetry protecting the Higgs mass in this model is an approximate $[\mathrm{SU}(3)/\,\mathrm{SU}(2)]^2$. We discuss the non-linear sigma model version of this theory and show how to get there from the linear sigma model.

In section 3 we present a complete model, including a quartic Higgs coupling, which has no quadratic divergences at one loop. Extending the gauge group to $\mathrm{SU}(4) \times \mathrm{U}(1)$ allows one to generate a quartic coupling through a vacuum misalignment mechanism. The non-linear sigma model is $[\mathrm{SU}(4)/\,\mathrm{SU}(3)]^4$. The result is a two Higgs doublet model with heavy $\mathrm{SU}(4)/\,\mathrm{SU}(2)$ gauge bosons and new scalars at a TeV.

In section 4 we discuss the spectrum of the SU(4) model and indicate some of the phenomenological constraints [10]–[15]. Because our model has no more gauge couplings than the SM, all couplings and masses in the gauge sector are a function of the breaking scale f and measured gauge couplings. We also comment on flavor issues in these models.

In the last section we discuss possibilities for future directions.

2. Little Higgs from a simple gauge group

In this section we describe a very simple extension of the SM which keeps the Higgs naturally light by employing the little Higgs mechanism. The model contains new particles and couplings which cancel the quadratic divergences from the top quark loop and from the SM gauge interactions. This is accomplished by enlarging the SU(2) weak gauge interactions to SU(3). The cancellation of the divergences from the Higgs self-coupling is more difficult and is described in section 4.

Our mechanism for eliminating the one-loop divergence from gauge interactions (figure 1(b) can be understood as follows: an SU(3) gauge group is spontaneously broken near 1 TeV by a vacuum expectation value (VEV) of two SU(3) triplet scalars. The triplet expectation values are aligned so that both vevs leave the same SU(2) unbroken. Ignoring their coupling trough the gauge interactions each scalar breaks a global SU(3) $\to$ SU(2), each yielding 5 Nambu-Goldstone bosons (NGBs). With the SU(3) gauge interactions turned on, the diagonal linear combination of NGBs is eaten, but the orthogonal linear combination remains massless at tree level. The masslessness of these modes is easily understood by noticing that in absence of a direct coupling between the two scalar triplets each of them "thinks" that it is the only field which breaks the gauge group and therefore contains exact NGBs. Quantum corrections from loops involving the gauge bosons generate couplings between the two triplets and therefore a mass for the pseudo-Nambu-Goldstone bosons (PNGB). However, there is no quadratically divergent one-loop diagram involving both scalar triplets. Finite and log-divergent diagrams contribute small scalar masses of order $g/4\pi\ f \sim 100\,\mathrm{GeV}$.

More concretely, consider an SU(3) gauge theory with two scalar fields transforming as (complex) triplets $\mathbf{\Phi}_i,\ i = 1, 2$, of the gauge group with a potential

$$\frac{\lambda^2}{2}\left(\mathbf{\Phi}_1^\dagger\mathbf{\Phi}_1 - f^2\right)^2 + \frac{\lambda^2}{2}\left(\mathbf{\Phi}_2^\dagger\mathbf{\Phi}_2 - f^2\right)^2 \tag{2.1}$$

which generates VEVs for the $\mathbf{\Phi}$ s. For simplicity, we assume equal couplings and VEVs for both triplets. This defines a linear sigma model with two global SU(3) symmetries acting on the two triplets. The spontaneous breaking $[\mathrm{SU}(3)]^2 \to [\mathrm{SU}(2)]^2$ yields five NGBs from each scalar. We now weakly gauge an SU(3) such that both scalars are triplets under the gauge symmetry. This explicitly breaks the $[\mathrm{SU}(3)]^2$ global symmetry to diagonal SU(3). After spontaneous symmetry breaking the five "diagonal" NGBs are eaten by the Higgs mechanism and the five orthogonal linear combinations are PNGBs. The symmetry which they correspond to — "axial" SU(3) — is explicitly broken by the gauge couplings. But since the tree level scalar potential respects both SU(3)'s the PNGBs remain massless at tree level.

We parametrize the scalars as

$$\mathbf{\Phi}_1 = e^{i\Theta_1/f}\begin{pmatrix} 0 \\ 0 \\ f+\rho_1 \end{pmatrix} = e^{i\Theta_{\mathrm{eaten}}/f}\ e^{i\Theta/f}\begin{pmatrix} 0 \\ 0 \\ f+\rho_1 \end{pmatrix}$$

$$\mathbf{\Phi}_2 = e^{i\Theta_2/f} \begin{pmatrix} 0 \\ 0 \\ f+\rho_2 \end{pmatrix} = e^{i\Theta_{\rm eaten}/f} e^{-i\Theta/f} \begin{pmatrix} 0 \\ 0 \\ f+\rho_2 \end{pmatrix} \tag{2.2}$$

where the first parametrization is the most obvious, but the second is more convenient because it separates the eaten modes $\Theta_{\rm eaten}$ from the PNGBs Θ. Note that $\Theta_{\rm eaten}$ shifts under diagonal (vector) SU(3) transformations whereas Θ shifts under "axial" SU(3). The ρ_i are radial modes which obtain masses $m_\rho = \lambda f$ from the potential. Since we are interested in the physics of the PNGBs we will suppress the eaten fields $\Theta_{\rm eaten}$. Furthermore we will take the limit $\lambda \to 4\pi$ in which the radial modes decouple and our linear sigma model turns into the corresponding non-linear sigma model. The non-linear sigma model obtained by integrating out the ρ_i includes non-renormalizable interactions which become strongly coupled at $\Lambda = 4\pi f$; at this scale the non-linear sigma model description breaks down. For most of this paper we will use the non-linear sigma model description because it is more general. It focuses on the physics of the PNGBs. Details of the UV theory which lead to the $[\mathrm{SU}(3)]^2$ symmetry breaking are encoded in higher dimensional operators and decouple from the relevant physics. The non-linear sigma model field are then parameterized as [16]

$$\Phi_1 = e^{i\Theta/f} \begin{pmatrix} 0 \\ 0 \\ f \end{pmatrix}, \qquad \Phi_2 = e^{-i\Theta/f} \begin{pmatrix} 0 \\ 0 \\ f \end{pmatrix} \tag{2.3}$$

where we suppressed the eaten fields and removed the bold type to distinguish non-linear from linear sigma model fields.

Expanded out in components, the five PNGBs in Θ are

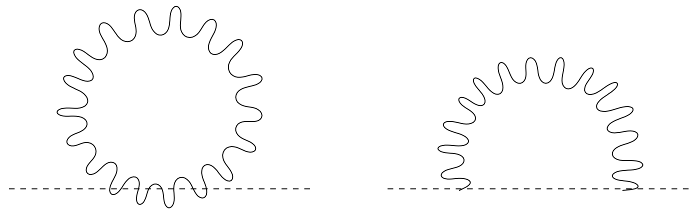
$$\Theta = \Theta^a T^a = \frac{1}{\sqrt{2}} \begin{pmatrix} \begin{matrix} 0 & 0 \\ 0 & 0 \end{matrix} & h \\ h^\dagger & 0 \end{pmatrix} + \frac{\eta}{4} \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & -2 \end{pmatrix}. \tag{2.4}$$

The T^a are the usual SU(3) generators, a runs from $4 \ldots 8$, and normalizations were chosen such that h and η have canonical kinetic terms. Under the unbroken SU(2) gauge symmetry h transforms like the SM Higgs, i.e. it is a complex doublet, and η is a neutral scalar.

Lets return to the linear sigma model. Since the gauge interactions explicitly break the axial SU(3) symmetry which protects the PNGBs we expect that quantum corrections from gauge interactions will generate a mass for them. In the linear sigma model the SU(3) gauge interactions are

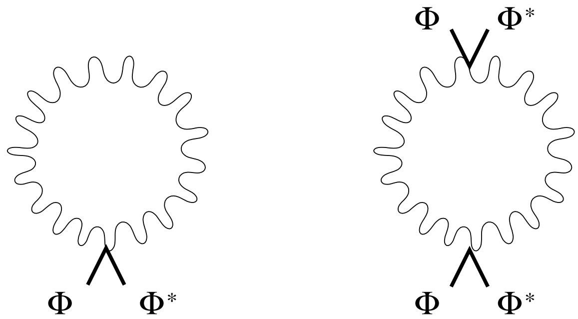
$$|(\partial_\mu + igA_\mu)\mathbf{\Phi}_1|^2 + |(\partial_\mu + igA_\mu)\mathbf{\Phi}_2|^2 . \tag{2.5}$$

At one loop, there are two quadratically divergent diagrams which contribute to scalar masses (figure 2). For the following arguments we choose $\partial_\mu A^\mu = 0$ gauge. This is convenient because diagrams involving the trilinear gauge couplings cannot contribute to the scalar potential (no derivatives) in this gauge. Thus the second diagram vanishes.



(a) (b)

Figure 2: SU(3) gauge boson loop corrections to the $\mathbf{\Phi}$ mass.



(a) (b)

Figure 3: Gauge boson contributions to the $\mathbf{\Phi}$ potential in $\partial_\mu A^\mu = 0$ gauge.

Diagrams contributing to the scalar potential are shown in figure 3. The first diagram is quadratically divergent, the second one is log-divergent, and diagrams with even more $\mathbf{\Phi}$ insertions would be finite. The first diagram gives

$$\triangle\mathcal{L} \sim -\frac{g^2\,\Lambda^2}{16\pi^2}\left(\mathbf{\Phi}_1^\dagger\mathbf{\Phi}_1 + \mathbf{\Phi}_2^\dagger\mathbf{\Phi}_2\right), \tag{2.6}$$

which preserves both SU(3) symmetries and renormalizes the $\mathbf{\Phi}_i$ potentials, eq. (2.1). Again, taking $\Lambda = 4\pi f$ this contribution is not larger than the tree level terms already present in the potential and does not destabilize the desired vacuum. Thus at the level of quadratically divergent diagrams, the PNGBs remain massless.

Log-divergent and finite diagrams do contribute to PNGB masses. For example, the second diagram in figure 3 generates

$$\triangle\mathcal{L} \sim \frac{g^4}{16\pi^2}|\mathbf{\Phi}_2^\dagger\mathbf{\Phi}_1|^2 \log\left(\frac{\Lambda^2}{f^2}\right) \sim -\frac{f^2}{16\pi^2}h^\dagger h\,. \tag{2.7}$$

This operator explicitly violates axial SU(3) and contains a mass for the PNGBs. However, the mass is of order $f/4\pi$ which is sufficiently small. Note that the mass-squared generated is positive, which means that this term stabilizes the vacuum with aligned expectation values for $\mathbf{\Phi}_1$ and $\mathbf{\Phi}_2$. The top quark loop which we discuss in the next section generates a negative mass-squared and thereby triggers electroweak symmetry breaking.

This analysis could have also been performed in the non-linear sigma model. The expression for the gauge couplings is identical to eq. (2.5) except that the $\mathbf{\Phi}_i$ are replaced by Φ_i. Loops like figure 3(a) generate $|\Phi_i^\dagger \Phi_i| = f^2$, a quadratically divergent contribution to the cosmological constant but no mass. Finite and log-divergent diagrams generate $|\Phi_2^\dagger \Phi_1|^2$ which does contain a Higgs mass of order 100 GeV if $f \sim 1$ TeV.

The absence of quadratic divergences can also be understood by noting that the diagram in figure 3(a) only involves one of the non-linear sigma model fields. Thus it is identical to the corresponding diagram in a theory with only one Φ. But in this theory Θ would be eaten by the Higgs mechanism, thus it cannot have any non-derivative couplings in the lagrangian. Therefore, only diagrams which involve both Φ's (figure 3(b)) can contribute to the Higgs mass but they necessarily involve more internal propagators and are not quadratically divergent.

2.1 Top Yukawa coupling

We now show that it is straightforward to add fermions and a top Yukawa coupling which does not upset the radiative stability of the Higgs mass. This can be done for both the linear and non-linear sigma models but we will only present the analysis of the non-linear model.

Since SU(2)-weak is embedded into an SU(3) gauge group, the top-bottom SU(2)-doublet is enlarged to a triplet $Q^T = (t, b, \chi)$. A mass of order f for the extra fermion χ in the triplet and Yukawa couplings for the top quark are generated from couplings to the Φ's

$$\mathcal{L}_{\rm top} = \lambda_1 \chi_1^c \Phi_1^\dagger Q + \lambda_2 \chi_2^c \Phi_2^\dagger Q\,, \tag{2.8}$$

where χ_i^c are Weyl fermions with the quantum numbers of the SU(2)-singlet component of the top quark.[1] The Yukawa couplings λ_i can be chosen real by redefining the phases of the χ^c. Expanding to first order in the Higgs h

$$\begin{aligned}\mathcal{L}_{\rm top} &= f(\lambda_1 \chi_1^c + \lambda_2 \chi_2^c)\chi + \frac{i}{\sqrt{2}}(\lambda_1 \chi_1^c - \lambda_2 \chi_2^c) h \begin{pmatrix} t \\ b \end{pmatrix} + \cdots & (2.9)\\ &= m_\chi\, \chi^c \chi - i\lambda_t\, t^c\, h \begin{pmatrix} t \\ b \end{pmatrix} + \cdots\,, & (2.10)\end{aligned}$$

where in the last step we diagonalized the mass matrix by finding the heavy and light linear combinations of the χ^c. We find a χ mass $m_\chi = \sqrt{\lambda_1^2 + \lambda_2^2}\, f$, and a top Yukawa coupling, $\lambda_t = \sqrt{2}\,\lambda_1 \lambda_2 / \sqrt{\lambda_1^2 + \lambda_2^2}$. To obtain a sufficiently large top mass both couplings λ_i must be of order one.

The absence of quadratic divergences to the Higgs mass at one loop from these interactions is again most easily understood by examining Feynman diagrams with external Φ's. The diagram in figure 4(a) is quadratically divergent but preserves both SU(3) sym-

[1] Note that despite superficial similarities between eq. (2.8) and the top-color see-saw [17] there is an important difference. Unlike the top-χ Higgs couplings in [17], the couplings in eq. (2.8) "collectively" break an SU(3) symmetry protecting the Higgs mass. This is necessary for the cancelation of quadratic divergences to occur, and this is why we obtain a top Yukawa coupling which does not require fine tuning of the Higgs mass.

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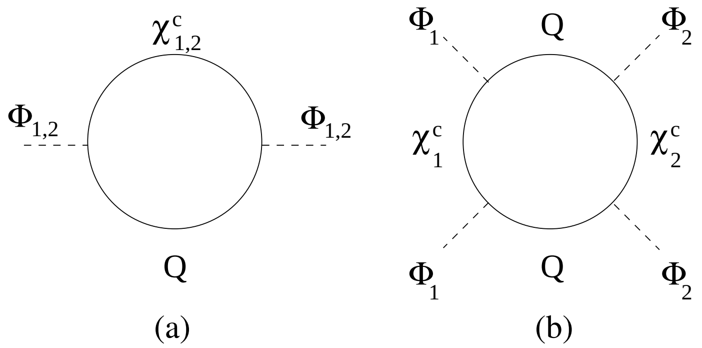

Figure 4: The top loop contribution to the Higgs mass in sigma model formalism.

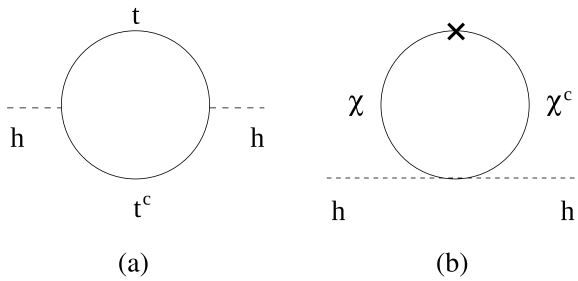

Figure 5: The canceling top and χ-loops in component form.

metries. Thus it does not contribute to PNGB masses. The diagram 4.b does contribute to the Higgs mass, but it is only log divergent, contributing to the operator $\lambda_1^2\lambda_2^2/16\pi^2|\Phi_2^\dagger\Phi_1|^2$ which contains a Higgs mass of order $f\lambda/4\pi$.

Alternatively, one can perform this computation after expanding the Φ's. In "component form" the vanishing of the quadratic divergence involves "miraculous" cancellations between loops of top quarks and loops of χ's. We demonstrate this calculation for the simplifying choice of $\lambda_1 = \lambda_2 \equiv \lambda_t$. Expanded out to the relevant order, the lagrangian contains

$$\lambda_t\left(\sqrt{2}f - \frac{1}{\sqrt{2}f}h^\dagger h\right)\chi^c\chi + \lambda_t t^c h \begin{pmatrix} t \\ b \end{pmatrix}. \tag{2.11}$$

In addition to the usual top loop there is also a quadratically divergent χ loop (figure 5). In the diagram the $\lambda_t\sqrt{2}f$ mass insertion on the χ line combines with the $-\lambda_t/\sqrt{2}f$ from the vertex to exactly cancel the quadratic divergence from the top loop. Note that although the cancellation involves a higher dimensional operator, its coefficient is related to the Yukawa coupling by the non-linearly realized axial SU(3) symmetry.

2.2 $[\mathrm{SU}(3)/\mathrm{SU}(2)]^2$ symmetry

Here we give an elegant way of understanding the absence of quadratic divergences in our theory which relies on the structure of the explicit breaking of the $[\mathrm{SU}(3)]^2$ symmetry acting on the Φ fields. The $[\mathrm{SU}(3)]^2$ symmetry not only protects both sets of NGBs (Θ_{eaten} and Θ) from obtaining a mass, it also forbids any non-derivative couplings of the NGBs. Therefore, in order to generate gauge- and Yukawa couplings for the Higgs, these symmetries must be explicitly broken. The trick is to do this breaking "non-locally in theory space" [3] or "collectively". By collective breaking we mean that no single coupling in the lagrangian breaks the symmetry by itself. Always at least two couplings are required to break any of the two SU(3)'s.

To see this explicitly, consider the gauge interactions

$$|(\partial_\mu + ig_1 A_\mu)\Phi_1|^2 + |(\partial_\mu + ig_2 A_\mu)\Phi_2|^2 , \tag{2.12}$$

where for clarity we have labeled the gauge couplings of Φ_1 and Φ_2 differently. Of course, gauge invariance requires them to have the same value, but as spurions they break different symmetries, and it is useful to keep track of them independently.

The key is to note that if either of the two couplings g_1 and g_2 is set to zero the lagrangian has an exact $[\mathrm{SU}(3)]^2$ symmetry, and therefore two sets of exact NGBs ($\Theta_{\rm eaten}$ and Θ) result from the spontaneous symmetry breaking. Only with both g's non-vanishing are the symmetries explicitly broken to the diagonal gauged SU(3). For $g_2 = 0$ we have the two independent symmetries

$$\Phi_1 \to U_1 \Phi_1 , \qquad A_\mu \to U_1 A_\mu U_1^\dagger , \qquad \Phi_2 \to U_2 \Phi_2 , \tag{2.13}$$

while for $g_1 = 0$ we have the symmetries

$$\Phi_1 \to U_1 \Phi_1 , \qquad A_\mu \to U_2 A_\mu U_2^\dagger , \qquad \Phi_2 \to U_2 \Phi_2 . \tag{2.14}$$

Thus when either of the g_i is set to zero Θ is an exact NGB. Any loop correction to its mass must be proportional to both g_1 and g_2. But, there are no quadratically divergent one loop diagrams involving both g_1 and g_2.

The argument for the Yukawa couplings is very similar. In absence of either λ_1 or λ_2 the Yukawa couplings in eq. (2.8) preserve two SU(3) symmetries. For example, when $\lambda_2 = 0$ we have

$$\Phi_1 \to U_1 \Phi_1 , \qquad Q \to U_1 Q , \qquad \Phi_2 \to U_2 \Phi_2 , \tag{2.15}$$

where Q denotes the quark triplet. Thus any Θ mass which is generated from Yukawa loops must be proportional to both λ_1 and λ_2. In fact, it must be proportional to $|\lambda_1|^2|\lambda_2|^2$. This follows from the two spurious $\mathrm{U}(1)_i$ symmetries under which only λ_i and χ_i^c are charged such that $\lambda_i \chi_i^c$ are neutral. Any operator which is generated by loops must also respect these spurious symmetries, and if it doesn't contain χ s it can only depend on the invariant combinations $|\lambda_1|^2$ and $|\lambda_2|^2$. Again, there is no quadratically divergent one-loop diagram proportional to $|\lambda_1|^2|\lambda_2|^2$.

2.3 Lets make the standard model

Three ingredients are still missing to turn the above model into a fully realistic extension of the Standard Model. We need (i) hypercharge and color, (ii) Yukawa couplings for the other fermions, and (iii) a quartic self coupling for the Higgs in order to stabilize its VEV.

(i) *Hypercharge and color:* adding color is trivial. To one loop the only relevance of color for the Higgs mass is a color factor of three in the top quark loop. Adding hypercharge is also straightforward. We introduce it by gauging a $U(1)_X$ symmetry which commutes with the SU(3) and under which the Φ_i have charge $-1/3$. In the non-linear sigma model $U(1)_X$ is non-linearly realized but the linear combination corresponding to hypercharge

$$Y = \frac{1}{\sqrt{3}} T_8 + \frac{1}{3} X \tag{2.16}$$

is unbroken. Here $T_8 = \frac{1}{\sqrt{3}} \operatorname{diag}(-\frac{1}{2}, -\frac{1}{2}, 1)$ is one of the broken SU(3) generators. The $U(1)_X$ charges of the fermions are uniquely determined from their hypercharges. For example, χ_i^c has charge $-\frac{2}{3}$ and Q has charge $\frac{1}{3}$.

Gauging $U(1)_X$ does not introduce new quadratic divergences for the Higgs mass. This is because $U(1)_X$ commutes with all the spurious global SU(3) symmetries which we used to argue for the absence of divergences. Thus these arguments go through unchanged.

(ii) *Yukawa couplings for light fermions:* since the Yukawa couplings of the other fermions are small, no care needs to be taken in their coupling to the Higgs in order to avoid large Higgs mass corrections. One possibility is to enlarge all left-handed fermion doublets into triplets and then write Yukawa couplings for up-type quarks and neutrinos just as we did for the top quark. For the bottom quark we can write

$$\frac{\lambda_b}{\Lambda}\, b^c \epsilon_{ijk} \Phi_1^i \Phi_2^j\, Q^k + \text{h.c.}\,, \tag{2.17}$$

and analogous terms for all down-type quarks and charged leptons. We postpone a more detailed discussion of flavor until section 4.

(iii) *Higgs quartic coupling:* one possibility for the quartic coupling is to ignore the numerically not very significant one-loop divergence from the Higgs loop and add a quartic coupling without canceling its divergence. A more satisfying model in which this divergence is also canceled requires more work. It is easy to appreciate the difficulty by looking at possible terms which one might add to the lagrangian to generate the quartic coupling. This would-be Higgs potential is an arbitrary gauge invariant polynomial in Φ_1 and Φ_2. The only non-trivial gauge invariant contraction at our disposal is $\Phi_1^\dagger \Phi_2$. The others either vanish ($\epsilon_{ijk} \Phi_1^i \Phi_1^j \Phi_2^k = 0$) or are constant ($\Phi_i^\dagger \Phi_i = f^2$). Thus the potential is a function of

$$\Phi_1^\dagger \Phi_2 = f^2 + i f \eta - h^\dagger h - \frac{1}{2}\eta^2 + \cdots + \frac{1}{6f^2}(h^\dagger h)^2 + \cdots \tag{2.18}$$

and it's hermitian conjugate. Focusing on the h-dependence, we see that the quartic coupling always comes accompanied by a mass term when expanding out a term like $(\Phi_1^\dagger \Phi_2)^n$. Setting the coefficient of the quartic to one, the mass is of order f which is much too large. Note that if the constant term in the expansion of $\Phi_1^\dagger \Phi_2$ were not

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there, one could generate a quartic by simply squaring eq. (2.18). This observation will be the key to constructing a quartic in section 4. Of course, it is possible to fine tune the mass term away. For example the potential $|\Phi_1^\dagger\Phi_2 - f^2|^2$ does not contain a mass term. But this is no better than the tuning of the Higgs mass in the Standard Model as the relative size of coefficients in $|\Phi_1^\dagger\Phi_2 - f^2|^2$ is not stable under quantum corrections.

3. A complete model: SU(4)

Now we present a complete model of electroweak symmetry breaking with no fine-tuning required to separate the Higgs mass from the cutoff. The main distinction from the previous model is the extension of the gauge group (and therefore the approximate global symmetries) from SU(3) to SU(4). The expansion of the group allows one to use a new mechanism to generate a quartic coupling.

Take Φ and Ψ as fields in the non-linear sigma model $[\mathrm{SU}(4)/\,\mathrm{SU}(3)]^2$ with the diagonal SU(4) gauged. The important distinction from the previous model is that the SU(4)-breaking is *not* aligned,

$$\Phi = e^{i\varphi/f}\begin{pmatrix}0\\0\\f\\0\end{pmatrix} \tag{3.1}$$

$$\Psi = e^{i\psi/f}\begin{pmatrix}0\\0\\0\\f\end{pmatrix} \tag{3.2}$$

and only the gauged SU(2) is linearly realized. Note that the product $\Phi^\dagger\Psi$ contains no constant term! This is exactly the success we were attempting to achieve in the SU(3) model. Raised to the appropriate power, it could potentially contain a term quartic in Higgses without a quadratic term.

This (mis-)alignment is stabilized when one adds the interaction $|\Phi^\dagger\Psi|^2$ to the potential with a positive coefficient. As we will see, this interaction becomes a positive squared mass for a charged scalar field — the only uneaten (complex) scalar in this example. The two SU(2) doublets which live in Φ and Ψ remain exact NGBs (they are in fact eaten) and thus this term does not induce a quartic term.

To reproduce the successes of the SU(3) model (and produce uneaten Higgs doublets), we break the gauged SU(4) $\to$ SU(2) twice. Our model has four sets of sigma model fields, $(\Phi_i, \Psi_i,\ i = 1, 2)$. Each contains one complex SU(2) doublet. Of the four doublets, two are eaten by the heavy SU(4) gauge bosons, leaving a two-Higgs-doublet model.

The complete counting goes as follows: the $[\mathrm{SU}(4)/\,\mathrm{SU}(3)]^4$ represents $(15-8)\times 4 = 28$ real components, 12 of which are eaten when the SU(4) gauge group is broken to SU(2). The remaining 16 consist of two complex doublets h_u and h_d, three complex SU(2) singlets

σ_1, σ_2 and σ_3, and two real scalars η_u and η_d. One possible parameterization is as follows:

$$\Phi_1 = e^{+i\mathcal{H}/f}e^{+i\Sigma_1/f}e^{+i\Sigma_2/f}e^{+i\Sigma_3/f}e^{+i\eta_u/f}\begin{pmatrix}0\\0\\f\\0\end{pmatrix} \tag{3.3}$$

$$\Phi_2 = e^{-i\mathcal{H}/f}e^{+i\Sigma_1/f}e^{-i\Sigma_2/f}e^{-i\Sigma_3/f}e^{-i\eta_u/f}\begin{pmatrix}0\\0\\f\\0\end{pmatrix} \tag{3.4}$$

$$\Psi_1 = e^{+i\mathcal{H}/f}e^{-i\Sigma_1/f}e^{+i\Sigma_2/f}e^{-i\Sigma_3/f}e^{+i\eta_d/f}\begin{pmatrix}0\\0\\0\\f\end{pmatrix} \tag{3.5}$$

$$\Psi_2 = e^{-i\mathcal{H}/f}e^{-i\Sigma_1/f}e^{-i\Sigma_2/f}e^{+i\Sigma_3/f}e^{-i\eta_d/f}\begin{pmatrix}0\\0\\0\\f\end{pmatrix} \tag{3.6}$$

with

$$\begin{aligned}
\mathcal{H} &= \frac{1}{\sqrt{2}}\begin{pmatrix}\begin{matrix}0&0\\0&0\end{matrix} & h_u & h_d\\ h_u^\dagger & 0 & 0\\ h_d^\dagger & 0 & 0\end{pmatrix} &\quad \Sigma_1 &= \frac{1}{2}\begin{pmatrix}0&0&0&0\\0&0&0&0\\0&0&0&\sigma_1\\0&0&\sigma_1^\dagger&0\end{pmatrix}\\
\Sigma_2 &= \frac{1}{2}\begin{pmatrix}0&0&0&0\\0&0&0&0\\0&0&0&\sigma_2\\0&0&\sigma_2^\dagger&0\end{pmatrix} &\quad \Sigma_3 &= \frac{1}{2}\begin{pmatrix}0&0&0&0\\0&0&0&0\\0&0&0&\sigma_3\\0&0&\sigma_3^\dagger&0\end{pmatrix}\\
\eta_u &= \frac{\eta_u}{6}\begin{pmatrix}1&0&0&0\\0&1&0&0\\0&0&-3&0\\0&0&0&1\end{pmatrix} &\quad \eta_d &= \frac{\eta_d}{6}\begin{pmatrix}1&0&0&0\\0&1&0&0\\0&0&1&0\\0&0&0&-3\end{pmatrix}.
\end{aligned} \tag{3.7}$$

A quartic term for this model will come from the four couplings

$$\kappa_{11}|\Phi_1^\dagger\Psi_1|^2 + \kappa_{22}|\Phi_2^\dagger\Psi_2|^2 + \kappa_{12}|\Phi_1^\dagger\Psi_2|^2 + \kappa_{21}|\Phi_2^\dagger\Psi_1|^2\,. \tag{3.8}$$

To leading order in each field, these operators produce the potential:

$$\kappa_{11}f^2|\sigma_1+\sigma_3|^2 + \kappa_{22}f^2|\sigma_1-\sigma_3|^2 + \kappa_{12}f^2\left|\sigma_1+\sigma_2-i\frac{h_u^\dagger h_d}{f}\right|^2 + \kappa_{21}f^2\left|\sigma_1-\sigma_2-i\frac{h_u^\dagger h_d}{f}\right|^2. \tag{3.9}$$

Any one coupling does not produce a potential for the Higgses. In fact, removing any of the couplings above removes the quartic Higgs term. To see this, note that there are other parameterizations in which the $h_u^\dagger h_d$ appear in different operators.

These couplings generate masses of order $\sqrt{\kappa} f$ for the complex scalars σ_i as well as trilinear terms marrying these scalars to $h_u^\dagger h_d$. Integrating out the singlets produces a quartic coupling:

$$\lambda = \frac{4}{\kappa_{11}^{-1} + \kappa_{22}^{-1} + \kappa_{12}^{-1} + \kappa_{21}^{-1}} \tag{3.10}$$

which vanishes as any one $\kappa_{ij} \to 0$, thus all four terms are required to produce a tree-level quartic term.

From symmetry arguments we see why this works. The non-linearly realized global symmetry of the model is approximately $[\mathrm{SU}(4)/\mathrm{SU}(3)]^4$ and contains, among other things, four Higgs-like doublets which are NGBs. Two of the doublets are eaten. A single operator, e.g., $|\Phi_1^\dagger \Psi_1|^2$, explicitly breaks the symmetry down to $[\mathrm{SU}(4)/\mathrm{SU}(3)]^2 \times [\mathrm{SU}(4)/\mathrm{SU}(2)]$, but this symmetry *also* contains four doublet NGBs. Adding $|\Phi_2^\dagger \Psi_2|^2$ leaves $[\mathrm{SU}(4)/\mathrm{SU}(2)]^2$ again producing four doublets. The existence of three of the four operators breaks enough symmetry to allow a quartic term, but only a small one is induced at loop level.

This structure also suppresses one-loop contributions to the Higgs mass. The symmetry arguments above show that more than one spurion is required to generate a Higgs potential, and therefore a Higgs mass. Thus, at one loop there are no quadratically divergent diagrams.

The gauge and top loops work similarly to those in the SU(3) model. The gauge loop is canceled for each Higgs doublet by the massive gauge bosons and the calculation goes through as in section 2. Gauging an additional U(1) symmetry results in the existence of hypercharge at the weak scale and does not contribute to the Higgs potential. The top Yukawa coupling could, for example, come from:

$$\mathcal{L}_{\text{top}} = \left(\lambda_1 \chi_1^c \Phi_1^\dagger + \lambda_2 \chi_2^c \Phi_2^\dagger + \lambda_3 \chi_3^c \Psi_1^\dagger\right) Q \tag{3.11}$$

where $Q^T = (t, b, \chi_1, \chi_2)$. The χ_3^c field is there to cancel the hypercharge anomaly and to marry and give a mass to χ_2. The λ_3 term does not play a role in generating a top Yukawa coupling and thus the coupling can be taken to the cutoff (i.e., $\lambda_3 \to 4\pi$) thus decoupling these extra fields and making the physics the same as the SU(3) case of the previous section. The bottom Yukawa coupling can be written as

$$\mathcal{L}_{\text{bottom}} = \lambda_b b^c \,\epsilon_{ijkl}\, \Phi_1^i \Psi_1^j \Psi_2^k Q^l \ . \tag{3.12}$$

This single coupling generates a quadratic divergence to the down-type Higgs mass of the form $|\Psi_1^\dagger \Psi_2|^2$. For a small bottom Yukawa coupling ($\lambda_b \lesssim 0.1$), the contribution remains at or below the weak scale.

3.1 Electroweak symmetry breaking

The purpose of the quartic term in the Higgs potential is to stabilize the Higgs VEV . The SU(4) model above has a quartic potential of the form

$$\mathcal{L}_{\text{quartic}} = -\lambda |h_u^\dagger h_d|^2 \tag{3.13}$$

where h_u and h_d are SU(2) doublets with hypercharge $Y = -1/2$. Successful electroweak symmetry breaking requires the existence of a mass term of the form $Bh_u^\dagger h_d$ where B is of order M_W^2. Such a term comes from operators $B_{11}\Phi_1^\dagger\Psi_1$, $B_{22}\Phi_2^\dagger\Psi_2$, $B_{12}\Phi_1^\dagger\Psi_2$, and $B_{21}\Phi_2^\dagger\Psi_1$, and can be written with the quartics as $\kappa_{11}|b_{11} + \Phi_1^\dagger\Psi_1|^2 + \kappa_{22}|b_{22} + \Phi_2^\dagger\Psi_2|^2 + \ldots$. These new operators contain linear terms for the uncharged scalar fields σ_i causing them to obtain VEVs. When the σ's are shifted to their minima, a mass term of the form $Bh_u^\dagger h_d$ is produced with

$$B = \frac{1}{2}\lambda \sum_{ij} b_{ij}\,. \tag{3.14}$$

Here — for simplicity — we have taken the b_i to be real, though in general their phases may have interesting implications for CP violation. These B terms are themselves spurions which explicitly break more symmetries than the quartic couplings. To see this, note that they are the only terms so far which produce a potential for the η fields. Thus their size, which needs to be of order $f^2/16\pi^2$ is technically natural, but undetermined from dynamics in the effective theory below Λ.

In addition, operators of the form $|\Phi_1^\dagger\Phi_2|^2$ and $|\Psi_1^\dagger\Psi_2|^2$ are generated by two-loop quadratic-divergent and one-loop log-divergent diagrams. They produce the mass terms

$$\mathcal{L}_{\text{mass}} = m_2^2|h_u|^2 + m_1^2|h_d|^2 \tag{3.15}$$

with a natural size of order $f^2/16\pi^2$. We require $m_2^2, m_1^2 > 0$ (or else the Higgs vev could run away to $\sim f$). Electroweak symmetry breaking occurs if

$$B > \sqrt{m_2^2 m_1^2}\,. \tag{3.16}$$

This Higgs potential is of the same form as the one in the SU(6)/ Sp(6) little Higgs model [7], and we repeat some of the phenomenology here. Formulas for the scalar masses in general two Higgs doublet models are conveniently collected in [18]. Minimizing the potential under these conditions gives $\tan\beta = v_u/v_d \equiv \langle h_u\rangle/\langle h_d\rangle = \sqrt{m_1^2/m_2^2}$ and

$$\frac{2B}{\sin 2\beta} = m_2^2 + m_1^2 + 2\lambda v^2 \tag{3.17}$$

where $v = 174\,\text{GeV}$ is the electroweak symmetry breaking scale.

The masses of the two CP-even Higgs bosons are

$$M_{h^0}^2, M_{H^0}^2 = \frac{B}{\sin 2\beta} \pm \sqrt{\frac{B^2}{\sin^2 2\beta} + \lambda v^2\left(\lambda v^2 - \frac{2B}{\sin 2\beta}\right)\sin^2 2\beta}\,. \tag{3.18}$$

The lightest CP-even Higgs boson is bounded from above by $M_{h^0}^2 \leq \lambda v^2$. This bound is saturated for $m_1^2 = m_2^2 \to \sin 2\beta = 1$. The CP-odd and charged Higgses have masses

$$M_{A^0}^2 = \frac{2B}{\sin 2\beta} \tag{3.19}$$

$$M_{H^\pm}^2 = M_{A^0}^2 - \lambda v^2 \tag{3.20}$$

3.2 The standard model embedding

Now we can construct a complete standard model based on the SU(4) theory. Collecting the pieces together, the lagrangian of the theory is

$$\mathcal{L} = \mathcal{L}_{\text{kinetic}} + \mathcal{L}_{\text{quarks}} + \mathcal{L}_{\text{leptons}} + \mathcal{L}_{\text{Higgs}} \tag{3.21}$$

with

$$\mathcal{L}_{\text{kinetic}} = |\mathcal{D}_\mu \Phi_i|^2 + |\mathcal{D}_\mu \Psi_i|^2 + [\text{fermion and gauge kinetic terms}] \ . \tag{3.22}$$

Here $\mathcal{D}_\mu = (\partial_\mu + i g_4 A^a_\mu T^a - i\frac{g_X}{4} A^X_\mu)$ and the 1/4 in the coupling of A^X_μ represents the $\mathrm{U}(1)_X$ charge of Φ and Ψ. A_μ and A^X_μ are the SU(4) and $\mathrm{U}(1)_X$ gauge fields respectively. The Yukawa couplings for quarks appear as

$$\mathcal{L}_{\text{quarks}} = \left(\lambda^u_1 \chi^c_{u1} \Phi^\dagger_1 + \lambda^u_2\ \chi^c_{u2} \Phi^\dagger_2 + \lambda^u_3 \chi^c_{u3} \Psi^\dagger_1\right) Q + \lambda^d d^c \Phi_1 \Psi_1 \Psi_2 Q \tag{3.23}$$

with $Q = (q, \chi_{u1}, \chi_{u2})^T$. We have suppressed flavor and SU(4) indices for clarity. The λ couplings are 3×3 matrices in flavor space — the combination of the first three produces the standard Yukawa matrix while λ^d is simply the Yukawa matrix for down-type quarks. Similarly, for the charged leptons:

$$\mathcal{L}_{\text{leptons}} = \left(\lambda^\nu_1 \chi^c_{\nu 1} \Phi^\dagger_1 + \lambda^\nu_2 \chi^c_{\nu 2} \Psi^\dagger_1\right) Q + \lambda^e e^c \Phi_1 \Psi_1 \Psi_2 L \tag{3.24}$$

where $L = (\ell, \chi_{\nu 1}, \chi_{\nu 2})^T$, we will discuss neutrino masses in the next section. Finally, the tree level scalar potential is

$$\mathcal{L}_{\text{scalar}} = \sum_{ij} \kappa_{ij} |b_{ij} + \Phi^\dagger_i \Psi_j|^2 \tag{3.25}$$

as discussed in the previous subsection.

Hypercharge is a linear combination of the SU(4) generator

$$T^{15} = \sqrt{2}\,\mathrm{diag}\left(-\frac{1}{4}, -\frac{1}{4}, +\frac{1}{4}, +\frac{1}{4}\right) \tag{3.26}$$

and the external $\mathrm{U}(1)_X$. Thus the X charges of the SU(4)-singlet fermions are just their respective hypercharges while the X charges of SU(4) vectors are the hypercharges of the SU(2) doublets they contain plus 1/4. Explicitly, (Φ_i, Ψ_i, L, Q) have X charges $(-1/4, -1/4, -1/4, +5/12)$.

4. Spectrum and constraints

A complete analysis of the phenomenology is beyond the scope of this paper but we would like to report on our initial explorations in this direction. The results are encouraging: we find significant constraints but there are large regions of parameter space which are in agreement with experiment while at the same time solving the hierarchy problem. Interestingly, the preferred region or parameter space will be directly explored at the Tevatron and LHC. More specifically, we will discuss (i) precision electroweak constraints (ii) the spectrum and direct searches (iii) flavor physics.

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(i) *Precision constraints:* one of the most stringent constraints on models of new physics at the TeV scale comes from isospin violating couplings of the light fermions to the W and Z bosons. One source of isospin violation is the different treatment of the Yukawa couplings for up and down-type quarks in eq. (3.23). Through the Higgs vev up-type quarks mix with the heavy χ fermions whereas down-type quarks don't. As we will now show, this mixing prefers different scales f_i for the different Φ_i.

For simplicity, we revert to our SU(3) model where the same mixing occurs. The up-type Yukawa couplings are

$$\left(\lambda_1 u_1^c \Phi_1^\dagger + \lambda_2 u_2^c \Phi_2^\dagger\right) \begin{pmatrix} u \\ d \\ \chi \end{pmatrix}, \tag{4.1}$$

where λ_i are 3×3 matrices in flavor space. In order to avoid large flavor changing effects (see FCNC discussion below) we take λ_2 proportional to the unit matrix and of order one whereas λ_1 is approximately equal to the usual Yukawa couplings of the up-type quarks in the Standard Model. Furthermore, we allow different scales f_1 and f_2 for the non-linear sigma models. The heavy SU(3) gauge bosons eat a linear combination of the NGBs which resides mostly in the sigma model with the larger scale, and the little Higgs lives mostly in the sigma model with the smaller scale:

$$\Phi_1 = e^{i\Theta \frac{f_2}{f_1}} \begin{pmatrix} 0 \\ 0 \\ f_1 \end{pmatrix}, \qquad \Phi_2 = e^{-i\Theta \frac{f_1}{f_2}} \begin{pmatrix} 0 \\ 0 \\ f_2 \end{pmatrix} \tag{4.2}$$

where

$$\Theta = \begin{pmatrix} \begin{matrix} 0 & 0 \\ 0 & 0 \end{matrix} & h \\ h^\dagger & 0 \end{pmatrix} / f_{12} \qquad \text{and} \qquad f_{12}^2 = f_1^2 + f_2^2 \,. \tag{4.3}$$

Substituting the Higgs by its expectation value $h^T = (v, 0)$ we obtain the mass matrix

$$\begin{pmatrix} u_1^c \\ u_2^c \end{pmatrix} \begin{pmatrix} \lambda_1 v \frac{f_2}{f_{12}} & \lambda_1 f_1 \\ -\lambda_2 v \frac{f_1}{f_{12}} & \lambda_2 f_2 \end{pmatrix} \begin{pmatrix} u \\ \chi \end{pmatrix} \tag{4.4}$$

Since $\lambda_2 \gg \lambda_1$ for the light quarks we see that the heavy quarks are approximately u_2^c, χ with masses $\lambda_2 f_2 \sim 1\,\text{TeV}$, and the light (SM) quarks are u_1^c, u with masses $\lambda_1 v f_2 / f_{12}$. In addition, there is small mixing between light and heavy quarks. The mixing between the u^c fields is not physical and can be removed by a change of basis. However, mixing between the SU(2) doublet component u and the singlet χ is significant because it alters the couplings of up-type quarks to the W and Z. The mixing angle is $\sim v f_1/(f_2 f_{12})$, which reduces the coupling of an up-type quark by

$$\delta g = -\frac{1}{2} \left(\frac{f_1 v}{f_2 f_{12}}\right)^2 . \tag{4.5}$$

For $f_1 \sim f_2 \sim 1\,\mathrm{TeV}$ and $v = 175\,\mathrm{GeV}$ the shift in the coupling is 1%. A similar shift also occurs in the couplings of neutrinos from their mixing with heavy partners. This is problematic because precision measurements at LEP and SLC have determined the gauge couplings of light fermions to a precision of $\sim 2 \times 10^{-3}$ [2]. However, we also see that it is easy to strongly suppress the mixing by taking $f_2 > f_1$. For example, taking $f_2 = 2\,\mathrm{TeV}$ and $f_1 = 1\,\mathrm{TeV}$ we have $\delta g \sim 10^{-3}$. We see that the part of parameter space with $f_2 > f_1$ is preferred.

We should check that taking unequal f_i allows a large enough top Yukawa coupling and does not destabilize the Higgs mass. Diagonalizing eq. (4.4) to leading order in v^2/f^2 for the third generation we obtain the mass of the heavy partner of the top and the top Yukawa coupling

$$m_\chi = \sqrt{\lambda_1^2 f_1^2 + \lambda_2^2 f_2^2}\,, \qquad \lambda_t = \lambda_1 \lambda_2 \sqrt{\frac{f_1^2 + f_2^2}{\lambda_1^2 f_1^2 + \lambda_2^2 f_2^2}} \tag{4.6}$$

Happily, a wide range of f_i and λ_i can give a heavy top quark without fine tuning of the Higgs mass. For example, taking $f_1 = .5\,\mathrm{TeV}$, $f_2 = 2\,\mathrm{TeV}$, $\lambda_1 = \sqrt{2}$ and $\lambda_2 = 1/3$ we obtain the correct top Yukawa. To estimate the degree of fine-tuning recall that the top loop contribution to the Higgs mass is cut off by m_χ. Thus $\delta m_h^2 \sim m_\chi^2\, \lambda_t^2/16\pi^2$ which requires no fine tuning for $m_\chi \simeq 1\,\mathrm{TeV}$.

We now turn to computing the masses and mixings of the gauge bosons in the full $\mathrm{SU}(4) \times \mathrm{U}(1)_X$ model. Transitions mediated by the heavy SU(4) gauge bosons contribute to precision electroweak measurements leading to constraints on the f_i. A useful parametrization of the non-linear sigma model fields Φ_i and Ψ_i with general f_i is

$$\begin{aligned} \Phi_1 &= e^{+i\mathcal{H}_u \frac{f_2}{f_1}} \begin{pmatrix} 0 \\ 0 \\ f_1 \\ 0 \end{pmatrix} \qquad & \Phi_2 &= e^{-i\mathcal{H}_u \frac{f_1}{f_2}} \begin{pmatrix} 0 \\ 0 \\ f_2 \\ 0 \end{pmatrix} \\ \Psi_1 &= e^{+i\mathcal{H}_d \frac{f_4}{f_3}} \begin{pmatrix} 0 \\ 0 \\ 0 \\ f_3 \end{pmatrix} \qquad & \Psi_2 &= e^{-i\mathcal{H}_d \frac{f_3}{f_4}} \begin{pmatrix} 0 \\ 0 \\ 0 \\ f_4 \end{pmatrix} \end{aligned} \tag{4.7}$$

where

$$\mathcal{H}_u = \begin{pmatrix} \begin{matrix} 0 & 0 \\ 0 & 0 \end{matrix} & h_u & \begin{matrix} 0 \\ 0 \end{matrix} \\ h_u^\dagger & 0 & 0 \\ \begin{matrix} 0 & 0 \end{matrix} & 0 & 0 \end{pmatrix} / f_{12} \qquad \mathcal{H}_d = \begin{pmatrix} \begin{matrix} 0 & 0 \\ 0 & 0 \end{matrix} & \begin{matrix} 0 \\ 0 \end{matrix} & h_d \\ \begin{matrix} 0 & 0 \end{matrix} & 0 & 0 \\ h_d^\dagger & 0 & 0 \end{pmatrix} / f_{34} \tag{4.8}$$

Here we ignore the small contributions to masses from small vevs for the fields σ_i. The photon and the Z are linear combinations of four neutral gauge bosons: three gauge bosons which correspond to the diagonal SU(4) generators $T^3 = \frac{1}{2}\,\mathrm{diag}(1, -1, 0, 0)$,

$T^{12} = \frac{1}{2}\,\mathrm{diag}(0,0,1,-1)$, and $T^{15} = \frac{1}{\sqrt{8}}\,\mathrm{diag}(-1,-1,1,1)$, and the $\mathrm{U}(1)_X$ gauge field B^x_μ. Two linear combinations obtain masses of order f from the kinetic terms of the Φ_i and Ψ_i

$$\begin{aligned} &\left|\frac{g}{2}A^{12}_\mu + \frac{g}{2\sqrt{2}}A^{15}_\mu - \frac{g_x}{4}B^x_\mu\right|^2 f_{12}^2 \\ &\left|-\frac{g}{2}A^{12}_\mu + \frac{g}{2\sqrt{2}}A^{15}_\mu - \frac{g_x}{4}B^x_\mu\right|^2 f_{34}^2 \end{aligned} \tag{4.9}$$

where g, g_x are the SU(4) and U(1) gauge couplings and $f_{ij}^2 = f_i^2 + f_j^2$. In the following we specialize to $f \equiv f_{12} = f_{34}$ for which the heavy gauge boson mass matrix simplifies. Then the two heavy eigenstates are

$$\begin{aligned} Z''_\mu &= A^{12}_\mu \\ Z'_\mu &= \frac{\sqrt{2}gA^{15}_\mu - g_x B^x_\mu}{\sqrt{2g^2+g_x^2}}, \end{aligned}$$

with masses $m_{Z''} = gf$ and $m_{Z'} = \frac{gf}{2}\sqrt{2+g_x^2/g^2}$. The two eigenstates which remain massless at this order are

$$\begin{aligned} W^3_\mu &= A^3_\mu \\ B_\mu &= \frac{g_x A^{15}_\mu + \sqrt{2}gB^x_\mu}{\sqrt{2g^2+g_x^2}}. \end{aligned} \tag{4.10}$$

The Z obtains its mass from the Higgs vevs $v = \sqrt{v_u^2+v_d^2}$. Ignoring mixing with the Z' the mass term is

$$v^2\left|\frac{g}{2}W^3_\mu - \frac{g_x}{2}\frac{1}{\sqrt{1+g_x^2/2g^2}}B_\mu\right|^2. \tag{4.11}$$

From this expression we can read off the standard model gauge couplings. We see that the SU(2) coupling of the standard model is equal to the SU(4) coupling g and — setting the coefficient of B_μ equal to $g'/2$ — we have

$$g' = \frac{g_x}{\sqrt{1+g_x^2/2g^2}}. \tag{4.12}$$

Deviations from the standard model arise in this model at order v^2/f^2 from mixing of the Z with the Z'. Explicitly, the mixing is determined by diagonalizing the Z–Z' mass matrix

$$\frac{g^2}{2}\begin{pmatrix} v^2(1+t^2) & -v^2(1-t^2)\sqrt{1+t^2}/\sqrt{2-t^2} \\ -v^2(1-t^2)\sqrt{1+t^2}/\sqrt{2-t^2} & 2f^2/(2-t^2) \end{pmatrix} \tag{4.13}$$

where $t = g'/g = \tan\theta_W$ and θ_W is the weak mixing angle. Diagonalizing, we find a contribution to $\delta\rho$ from the shift in the Z mass

$$\delta\rho \equiv \frac{\delta m_W^2}{m_W^2} - \frac{\delta m_Z^2}{m_Z^2} = \frac{v^2}{2f^2}(1-t^2)^2 \approx +1.5\cdot 10^{-3}\left(\frac{2.2\,\mathrm{TeV}}{f}\right)^2. \tag{4.14}$$

Given that a standard model fit predicts a W-mass which is lower than the experimental value by about 1.6σ [2], this correction actually improves the precision electroweak fit for $f \sim 2.2\,\mathrm{TeV}$. Alternatively, demanding a fit that is at least as good as the standard model implies a bound of $f \gtrsim 1.5\,\mathrm{TeV}$.

Another observable affected by the new gauge bosons are four-fermion operators. The bound on new contributions to the four-electron operator, for example, is quite severe. The exchange of the Z' produces an operator of the size:

$$\frac{(1-t^2)^2}{8f^2}\bar{e}\gamma^\mu e\bar{e}\gamma_\mu e \tag{4.15}$$

for left-left currents. Using current bounds on this operator we find the requirement that $f \gtrsim 1.5\,\mathrm{TeV}$.

(ii) *The spectrum and direct searches:* in the UV the standard model $\mathrm{SU}(2)\times\mathrm{U}(1)$ gauge group is enlarged to $\mathrm{SU}(4)\times\mathrm{U}(1)$. Thus there are 12 new massive gauge bosons with masses near a TeV. Two of them are the Z' and Z'' discussed above. For the parameter choice $f_1^2+f_2^2=f_3^2+f_4^2\equiv f^2$ the Z'' has mass gf and does not couple to standard model fermions. The Z' has mass $.77gf$ and couples to quarks and leptons. At the Tevatron it would appear as an s-channel resonance which decays to pairs of leptons. The limit on the mass of such a Z' from CDF [19] is in the 700-800 GeV range, implying a bound $f \gtrsim 1\,\mathrm{TeV}$. The off-diagonal $\mathrm{SU}(4)$ gauge bosons and their masses are

$$\begin{pmatrix} & & Y^0 & Y'^0 \\ & & X^- & X'^- \\ \bar{Y}^0 & X^+ & & Y''^0 \\ \bar{Y}'^0 & X'^+ & \bar{Y}''^0 & \end{pmatrix} = \begin{pmatrix} & & .5 & .5 \\ & & .5 & .5 \\ .5 & .5 & & 1 \\ .5 & .5 & 1 & \end{pmatrix} gf \tag{4.16}$$

The Y''^0 only couples to the heavy fermions and is therefore extremely difficult to detect. All others couple to one light and one heavy fermion. They can be produced in association with a heavy fermion or else appear in t-channel diagrams.

There are two vector-like heavy quarks of charge 2/3 for each generation. As we discussed above, one of them mixes with up-type quarks and can be produced singly in t-channel W exchange. The LHC reach in this channel can be as large as several TeV [15]. The masses of these quarks are not completely determined because they depend on unknown Yukawa couplings. But flavor constraints suggest that their masses are generation independent and since naturalness requires a partner for the top quark below $\sim 2\,\mathrm{TeV}$ we expect at least one set of these quarks to be visible at the LHC. In terms of model parameters, the new quark masses are $\sqrt{\lambda_1^2 f_1^2+\lambda_2^2 f_2^2}$ and $\lambda_3 f_3$.

In addition, there are also vector-like heavy leptons which mix with the neutrinos. These fermions are impossible to discover directly but their existence might be inferred by missing energy signals or through their mixing with the light neutrinos in precision data. Note also that this mixing reduces neutral currents of neutrinos more than charged currents which might help to explain the NuTeV anomaly [20] and the slightly reduced invisible width of the Z.

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The scalar spectrum consists of the two Higgs doublets with masses near the weak scale, three complex neutral fields σ_i with masses $\sim \kappa f$, and two real scalar fields η_i with masses of order the weak scale. The η_i only couple to heavy fields and in the b-terms of the Higgs potential, thus they are very difficult to detect despite their relatively low masses.

(iii) *Flavor:* the SU(4) model has more Yukawa matrices than the standard model. In general these additional sources of flavor violation lead to flavor changing neutral currents. The easiest way to suppress the flavor violation is to assume that the new Yukawa matrices are proportional to the unit matrix. This assumption imposes constraints on the UV completion of the theory, but it is "technically natural" in the effective theory (loops in the effective theory only generate small corrections). Note that this is similar to the assumption of universal soft masses in supersymmetry.

For example, the quark Yukawa couplings are then

$$\mathcal{L}_{\text{quarks}} = \left(\lambda^u u_1^c \Phi_1^\dagger + I_1 u_2^c \Phi_2^\dagger + I_2 u_3^c \Psi_1^\dagger\right) Q + \lambda^d d^c \Phi_1 \Psi_1 \Psi_2 Q \tag{4.17}$$

where λ^u and λ^d are similar to the usual standard model Yukawa couplings and I_1 and I_2 are approximately proportional to the unit matrix (in flavor space). To see that these couplings do not contain dangerous flavor violation, we go to a new basis in which λ^u is diagonal. It is convenient to rotate all four components of Q in the same way. I_1 and I_2 remain unit matrices if u_2^c and u_3^c are rotated appropriately. Finally, we also diagonalize λ^d with a bi-unitary transformation, but this time we transform only the down-type quarks in Q. In the new basis all Yukawa couplings are diagonal; flavor violation resides only in the gauge couplings of W, X, Y to quarks and in couplings of multiple Higgses to quarks which arise from expanding out the down Yukawa operator. The latter couplings are small and only appear in loops. The former are also easily shown to be harmless. There are the usual W couplings proportional to the CKM matrix in addition to new couplings between one down-type quark, one heavy vector-like up-type quark and the heavy gauge bosons X and Y. These couplings allow box diagrams and penguins which are similar to corresponding standard model diagrams with Ws replaced by Xs or Ys. The resulting flavor changing neutral currents are suppressed relative to the standard model ones by the large masses of the heavy gauge bosons m_W^2/m_X^2 and can be ignored.

A similar analysis of the lepton sector shows that there is also no dangerous lepton flavor violation in the low energy theory. Of course, the theory may also contain direct flavor violating four Fermi operators suppressed by the cut-off Λ. Such operators are constrained by $K - \overline{K}$ mixing and CP violation. The experimental bounds on such operators therefore imply constraints on the unknown UV-theory above 10 TeV.

Small neutrino masses can be obtained by including a higher dimensional lepton number violating operator $(\Phi^\dagger L)^2$ with a small coefficient in the effective theory. This operator might arise from a generalization of the see-saw mechanism in the UV completion: supermassive right handed neutrinos coupled to the operator which interpolates $(\Phi^\dagger L)$ in the UV theory.

5. Discussion

We have seen that, in a little Higgs model, embedding $\mathrm{SU}(2)_{weak}$ into a simple group (such as SU(3)) is enough to cancel one-loop quadratic divergences from gauge and (perturbatively coupled) fermion loops. The SU(3) model only lacks a quartic. One possibility is to simply ignore the relatively insignificant fine-tuning from the Higgs couplings and add the quartic by hand in "component" fields (i.e., not the full Φ). This coupling need not be very large as an additional contribution to the quartic would come from the log-divergent and finite contributions to the effective potential from the top sector below the scale f. At worst, this introduces of order 10% fine-tuning.

There are at least three different possibilities for ultra-violet completions to our models. One is a linear sigma model with supersymmetry protecting the scalar masses above the multi-TeV scale. The SU(3) model would work well in this case as the quartic would be provided by the D-term and the Higgs would remain massless at tree-level as long as there are more than two triplets. In addition, the group is simple enough to embed in a unifying theory: $\mathrm{SU}(3+n)_{\mathrm{color}} \times \mathrm{SU}(2+n)_{\mathrm{weak}} \times \mathrm{U}(1)$ gives coupling constant predictions extremely close to those of the MSSM, when charges are normalized to embed into $\mathrm{SU}(5+2n)$, and matter is in complete representations except for a split fundamental and anti-fundamental [21].

If one wishes to complete the theory above $4\pi f$ with a strongly coupled theory, the coset space we've used would require something different than a QCD-like model. For example, a gauged SU(7) with four fundamentals, one anti-fundamental and one anti-symmetric tensor produces the symmetry-breaking pattern $\mathrm{SU}(4) \rightarrow \mathrm{SU}(3)$ assuming the fundamentals condense with the anti-fundamental. Fermion and quartic interactions would require additional dynamics as in extended-technicolor [22].

A more interesting possibility would be a linear sigma model completing into another little Higgs theory at a higher scale, $F \sim 10\,TeV$. This may be possible in the SU(3) theory if the "fundamental" quartic added need not be too large. One example would be $[\mathrm{SU}(7)/\,\mathrm{SO}(7)]^2$ with SU(3) subgroups gauged similar to the "littlest Higgs" [5]. This could produce two light SU(3) triplets with fermion couplings causing vacuum misalignment and SU(3)-breaking at a scale $f \equiv F/4\pi$. If a model of this type works, it would provide a weakly coupled theory of electroweak symmetry breaking valid up to >100 TeV.

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